THE HOPF-LAPLACE EQUATION

(Joint work by Jan Cristina, Tadeusz Iwaniec, Leonid Kovalev and Jani Onninen)

Abstract

The central theme in this lecture is the Hopf-Laplace equation, which represents stationary solutions with respect to the inner variation of the Dirichlet integral. Among such solutions are harmonic maps. Nevertheless, minimization of the Dirichlet energy among homeomorphisms often leads to nonharmonic solutions. We investigate the Hopf-Laplace equation for a certain class of topologically well behaved mappings which are almost homeomorphisms, called ***Hopf deformations***. We establish Lipschitz continuity of Hopf deformations, the best possible regularity one can get. Thus in particular we show that the minimal-energy deformations are Lipschitz continuous, a result of considerable interest in the theory of minimal surfaces, calculus of variations, PDEs with potential applications to elastic plates.