

HARMONIC MAPPINGS AND THE NITSCHKE CONJECTURE

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Abstract. The Nitsche conjecture is deeply rooted in the theory of doubly connected minimal surfaces. However, it is commonly formulated as a question of existence of a harmonic homeomorphism between circular annuli

$$h : \mathbb{A} = A(r, R) \xrightarrow{\text{onto}} A(r_*, R_*) = \mathbb{A}^*$$

In the early 1960s, while attempting to describe all doubly connected minimal graphs over a given annulus \mathbb{A}^* , J.C.C. Nitsche observed that their conformal modulus cannot be too large. Then he conjectured, in terms of isothermal coordinates, even more;

A harmonic homeomorphism $h : \mathbb{A} \xrightarrow{\text{onto}} \mathbb{A}^$ exists if and only if:*

$$\frac{R_*}{r_*} \geq \frac{1}{2} \left(\frac{R}{r} + \frac{r}{R} \right)$$

This fascinating and engaging problem remained open for a half of a century. In this lecture I will present recent joint work with **Leonid Kovalev and Jani Onninen** in which we give, among further generalizations, an affirmative answer to his conjecture.