

# Aspects of regularity theory for minimizers of multiple integrals

Jan Kristensen

There are by now several examples of regular variational problems that admit singular minimizers (non-differentiable Sobolev maps, and worse) in the multi-dimensional vectorial case. This is in line with what one can prove: minimizers are *partially regular*, meaning they are smooth outside a small relatively closed subset of their domain. We refer to this set of singularities as the *singular set*. General measure theory easily gives that the singular set has zero Lebesgue measure, but over the years much better estimates of their size, in particular in terms of Hausdorff measures, have been derived. This is often done by proving that minimizers admit higher order weak derivatives. Still the gap between the examples provided by singular minimizers and the theoretical bounds for the size of the singular sets remains a major challenge.

In these lectures we start by briefly reviewing the known examples of singular minimizers and some related background results. We then discuss recent progress on the problem of higher differentiability of minimizers in both the convex and nonconvex cases.