A stability criterion for two fluids interfaces and applications

David Lannes

October 21 and 25, 2010

Abstract

These lectures are devoted to the behavior of the interface between two fluids of different densities. Kelvin-Helmholtz instabilities are known to destabilize such interfaces. The aim of these lectures is to derive a criterion that ensures that the instabilities can be controlled, thus explaining the observation of interfacial waves in various physical situations.

We will begin with an introduction on the one-fluid problem (water waves) and a quite formal description of the mechanisms that create Kelvin-Helmholtz instabilities in the two fluids case. We will then study in details the two fluid equations and exhibit the above mentioned stability criterion, which is a generalization of the Rayleigh-Taylor criterion for the water waves, or the linear Kelvin criterion. In particular, we will explain an apparent paradox: surface tension is necessary for the wave to exist but does not play any role on its propagation. In the last part of these lectures, we will be interested in application to physical situations and in particular to so called shallow water regimes. We will explain how to derive a "practical version" of the stability criterion very easy to check and focus on some specific difficulties linked to the shallow water limit. Applications will then be shown.